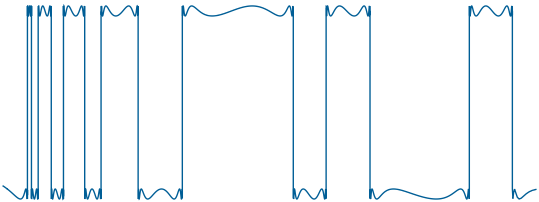
Multiband filters with optimal magnitude responses

November ?, 2015



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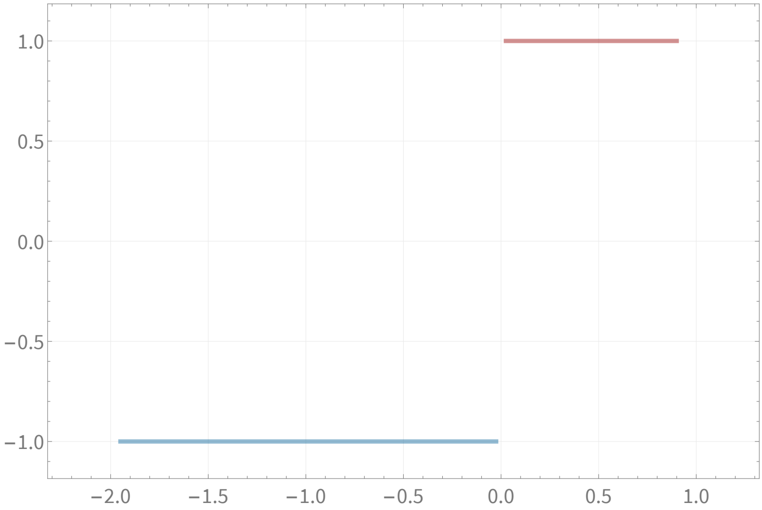
Zolotarev-Cauer (elliptic) filter

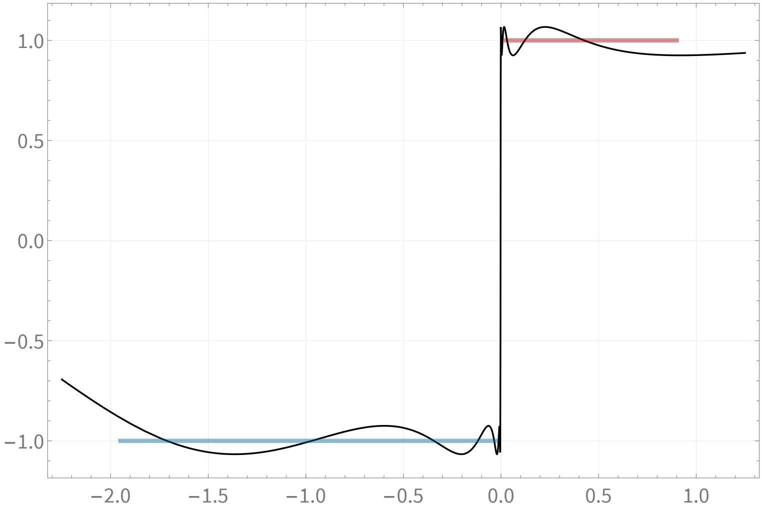
solved optimization problem in 1870s

applied Zolotarev's result in filters theory in 1930s, thereby inventing elliptic filters

Sign function:



Zolotarev fraction:



Zolotarev problem

Zolotarev problem: find the best rational approximant $R_n(w)$ of the given degree n to the transition function $F(w)$ in the uniform metric on E.

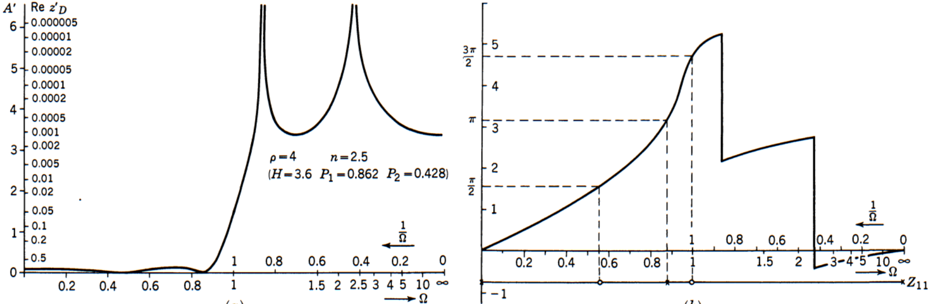
$$\max_{w \in E} |R_n(w) - F(w)| \;\to\; \min$$

$E = E_1 \cup E_2$ — set of two disjoint intervals

$F(w) = \begin{cases} -1, & w \in E_1 \\ 1, & w \in E_2 \end{cases}$ — transition function

Solution: Zolotarev fraction

Magnitude and phase responses of 4th order **elliptic filter** (images were scanned from Cauer's "Theorie der linearen wechselstrom schaltungen")



(a) (b)

Elliptic LP and HP filters are **optimal:**

- → Elliptic filters provide the **best approximation** of ideal low-pass or high-pass response among all filters of the same order.
- → The **order** of elliptic filter that is required to achive the given specification **is lower** than that of any other filter.

Generalized Zolotarev problem

transition function

$$\max_{w \in E} |R_n(w) - F(w)| \to \min$$

rational approximant

E is a set of $M \geqslant 2$ disjoint intervals

$F(w)$ is equal to 0 (stopband) or 1 (passband) on each interval of E

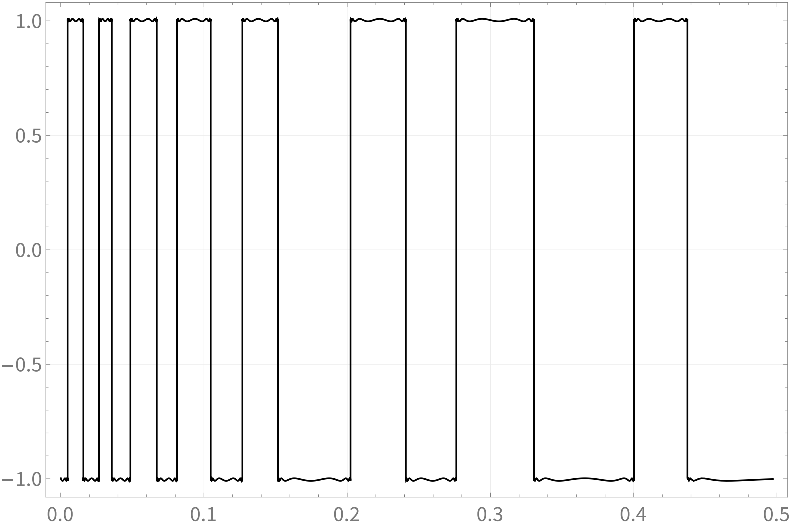
There were no methods for finding explicit solutions of **generalized Zolotarev problem** (when the number of intervals is greater than two) ...

There were no methods for finding explicit solutions of **generalized Zolotarev problem** (when the number of intervals is greater than two) ...

... until one was proposed in a paper

A.B. Bogatyrev, "*Chebyshev representation for rational functions*", Sb. Math., 201:11 (**2010**), 1579–1598.

Example of solution of generalized Zolotarev problem (17 intervals)



solutions of **Zolotarev problem**

↓

elliptic filters (optimal **low-pass and high-pass** filters)

solutions of **generalized Zolotarev problem**

↓

optimal **multiband** filters

New approach is based on using solutions of generalized problem and allows one to design optimal multiband filters.

Advantages:

→ Optimal filters provide the best approximation of given ideal multiband magnitude response among all stable filters of the same order.

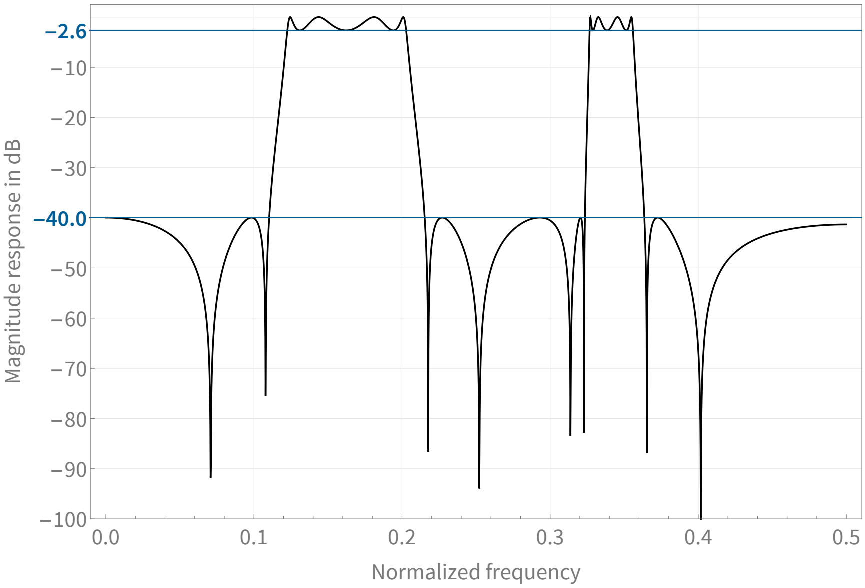
→ The order of optimal multiband filter that is required to achive the given specification is lower than that of filter designed by using any other method.

→ New approach works well when the number of passbands is up to 8 and the resulting filter's order is up to 500.

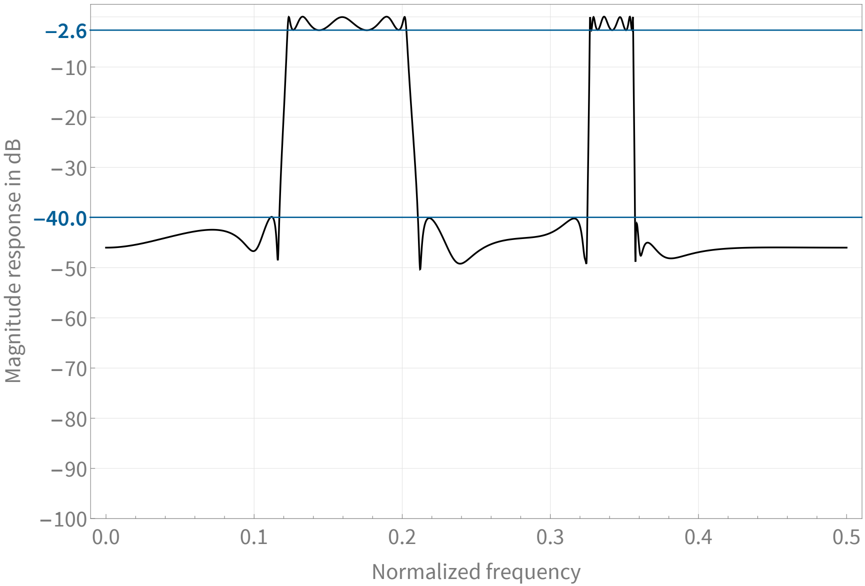
→ Even elliptic bandpass filters are not optimal. New approach allows one to design optimal bandpass filters.

DESIGN EXAMPLES

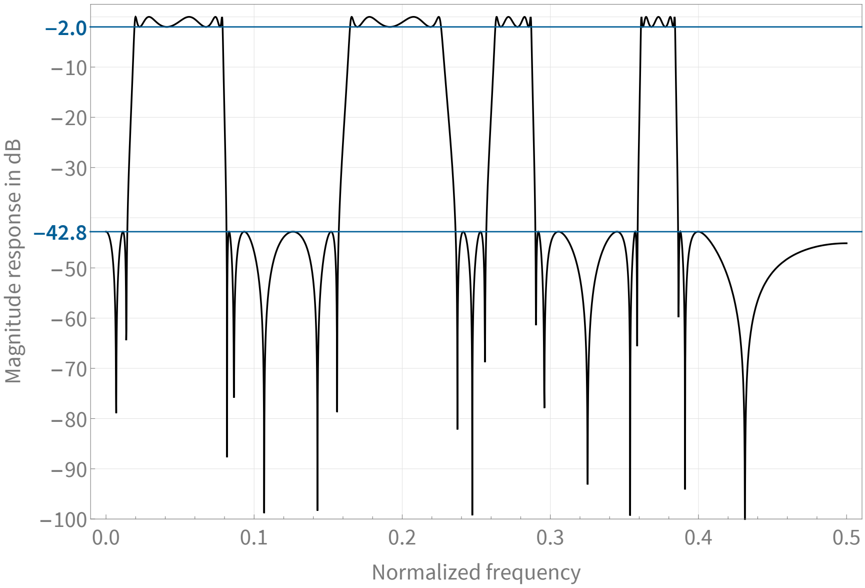
Magnitude response of 16 th order optimal filter



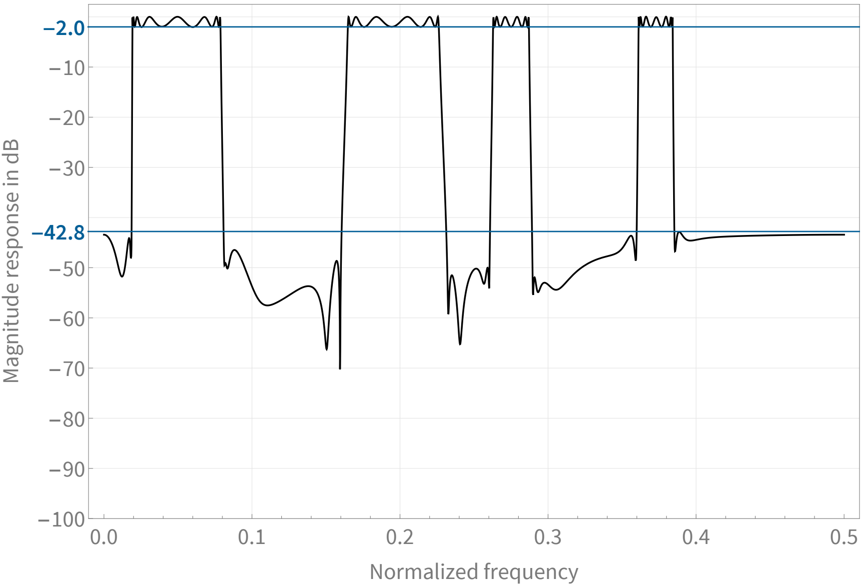
Magnitude response of 23th order composite filter



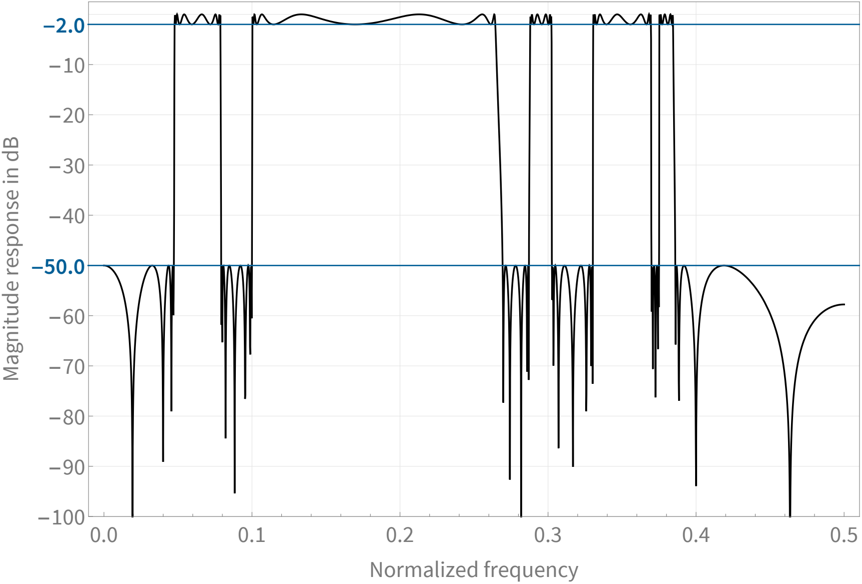
Magnitude response of 36th order optimal filter



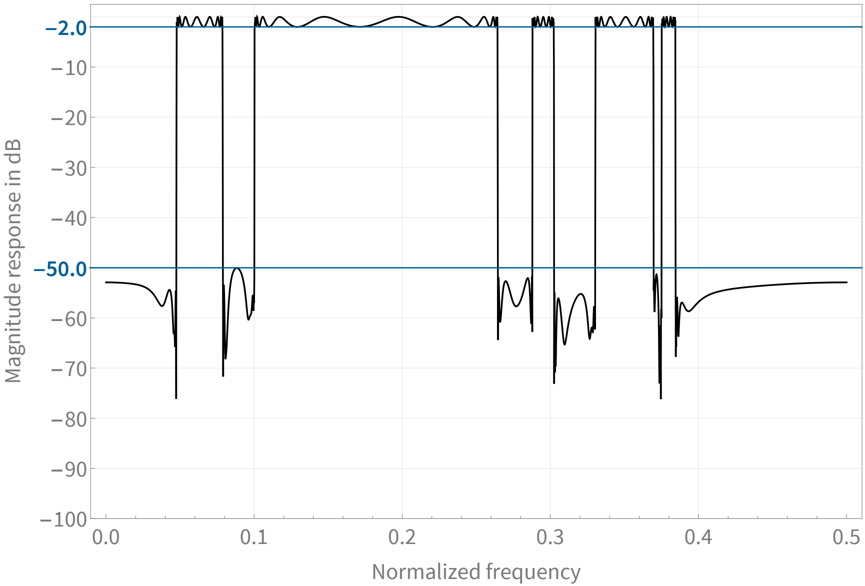
Magnitude response of 55th order composite filter



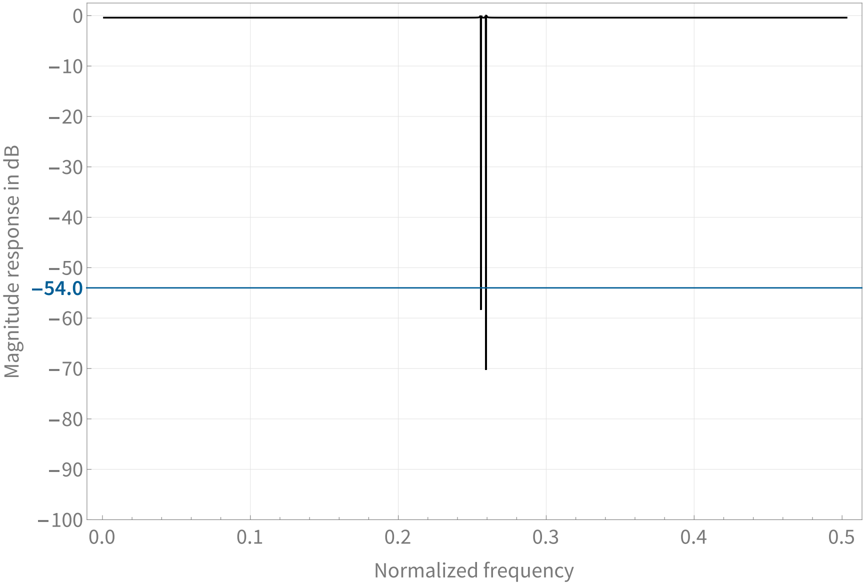
Magnitude response of 76 th order optimal filter



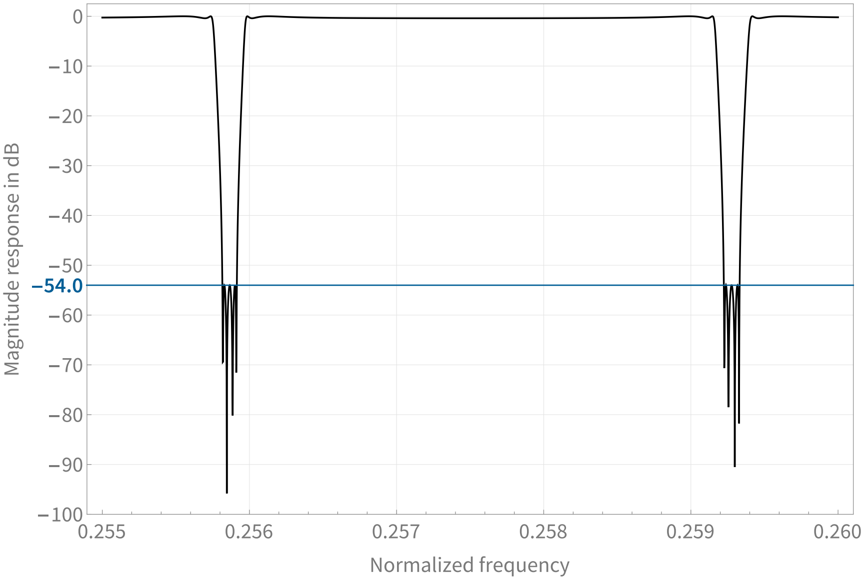
Magnitude response of 132th order composite filter



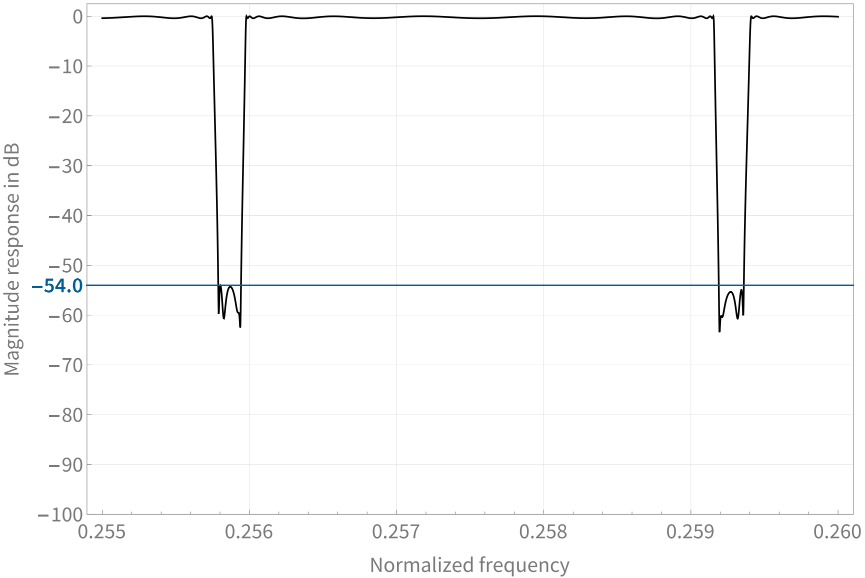
Magnitude response of 16th order optimal filter



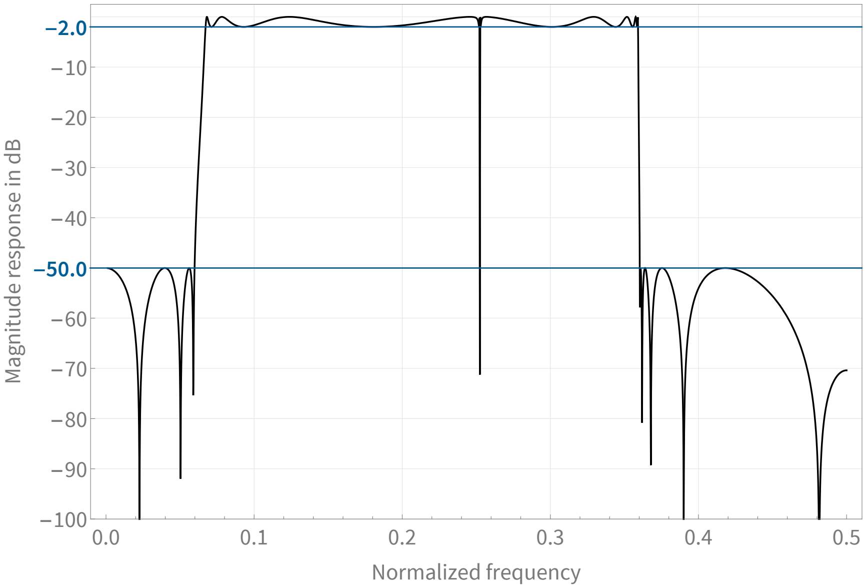
Magnitude response of 16th order optimal filter (zoomed)



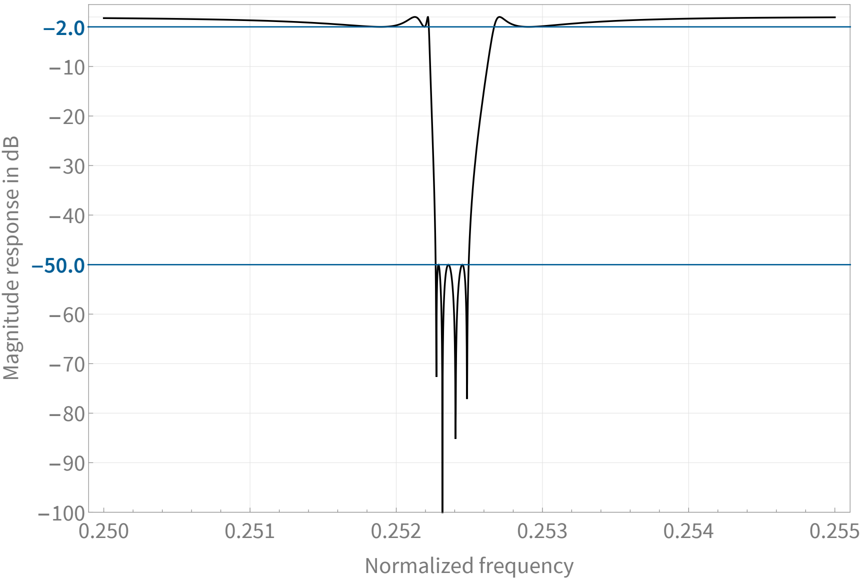
Magnitude response of 62th order composite filter (zoomed)



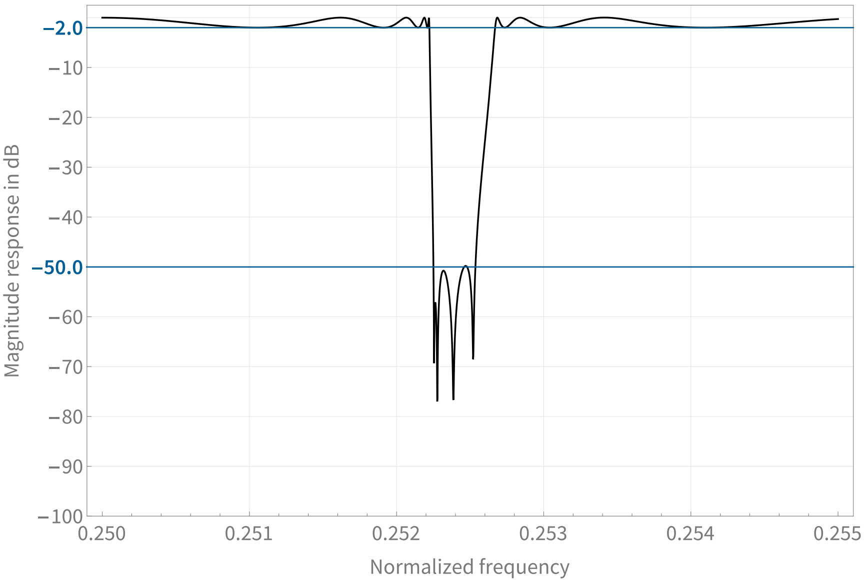
Magnitude response of 24 th order optimal filter



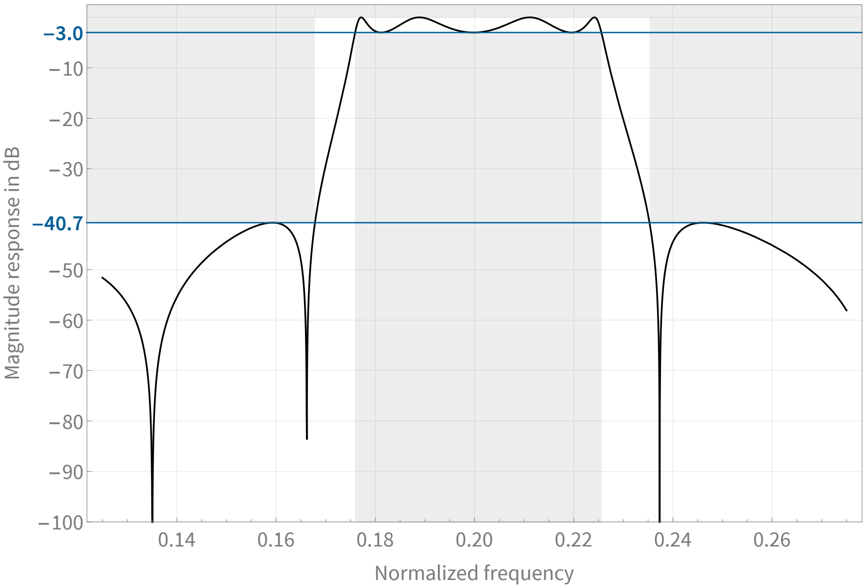
Magnitude response of 24th order optimal filter (zoomed)



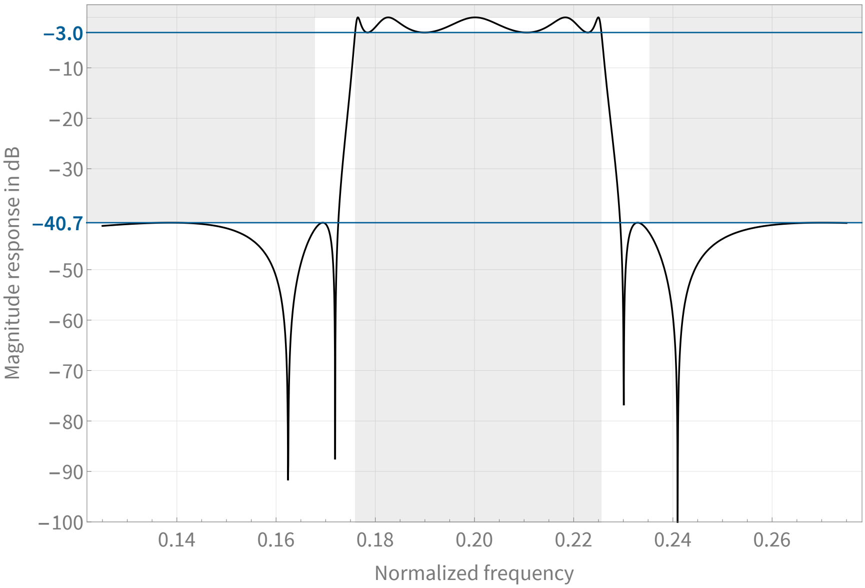
Magnitude response of 59 th order composite filter (zoomed)



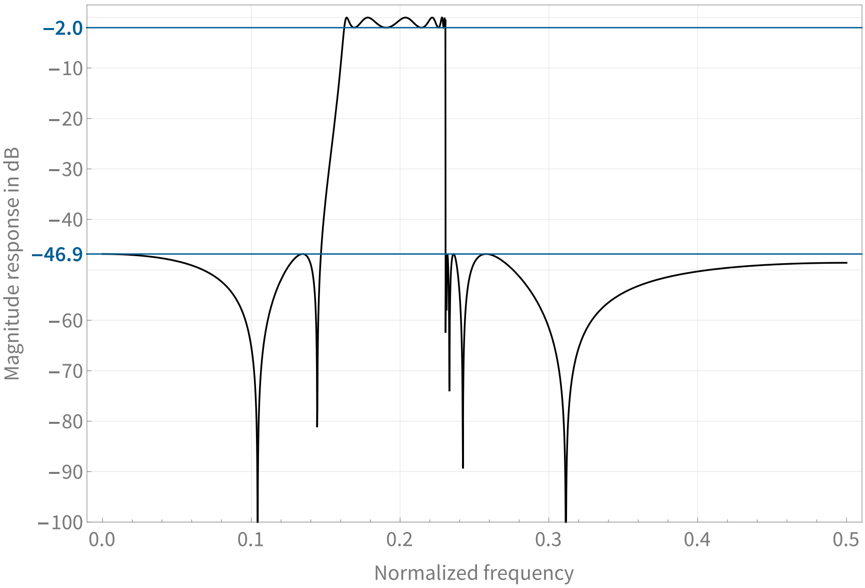
Magnitude response of 8th order optimal filter



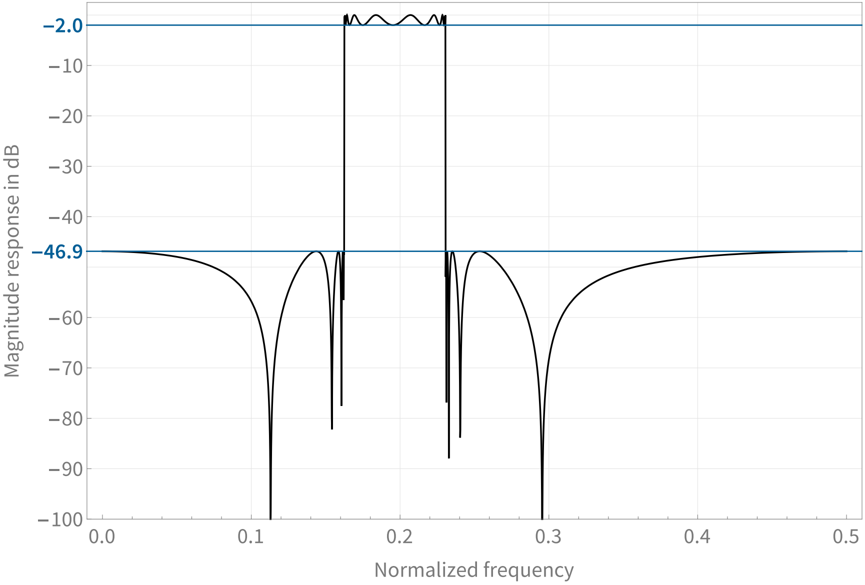
Magnitude response of 11th order elliptic filter



Magnitude response of 18th order optimal filter



Magnitude response of 28th order elliptic filter



Example	Optimal	Composite	Difference
2 passbands	16	23	7 (30 %)
4 passbands	36	55	19 (35 %)
5 passbands	76	132	56 (42 %)
notch (2 stopbands)	16	62	46 (74 %)
1 passband + notch	24	59	35 (60 %)
1 passband	8	11	3 (27 %)
1 passband	18	28	10 (26 %)

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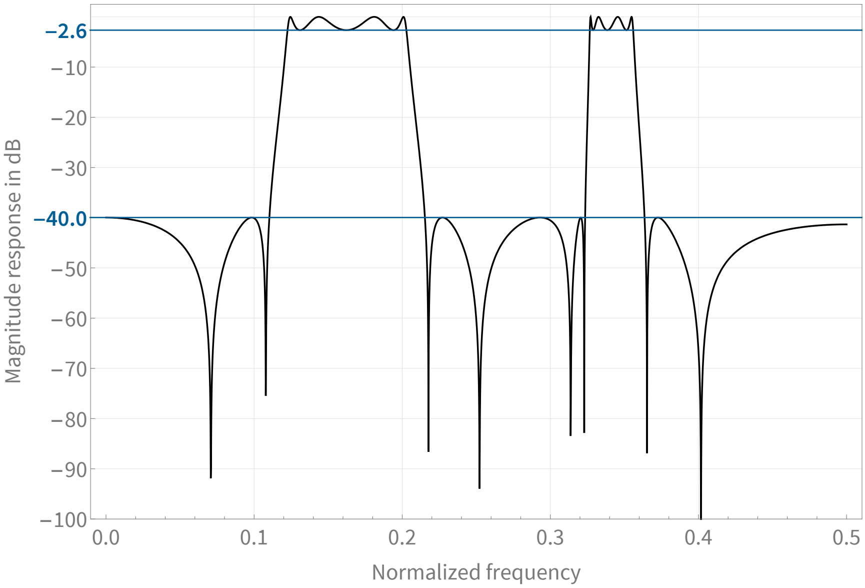
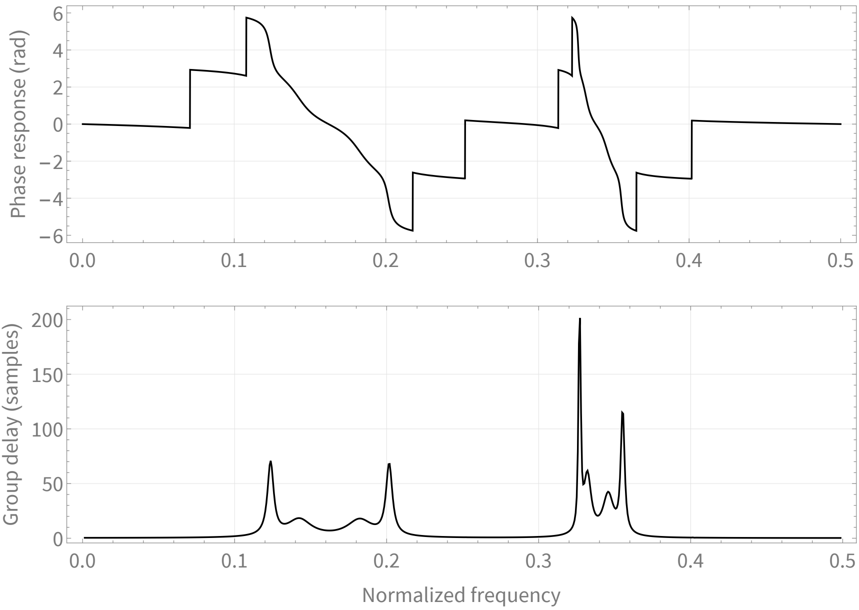
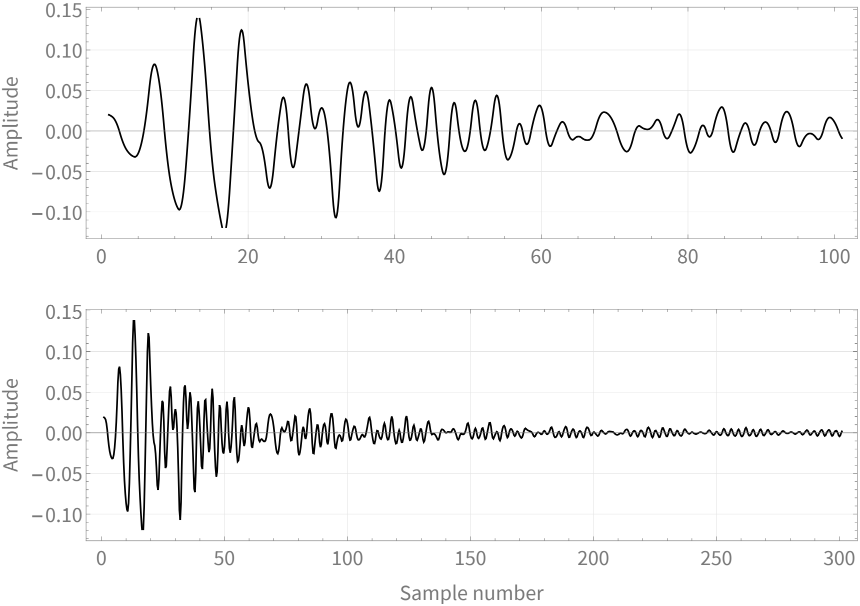
Magnitude response of 16 th order optimal filter



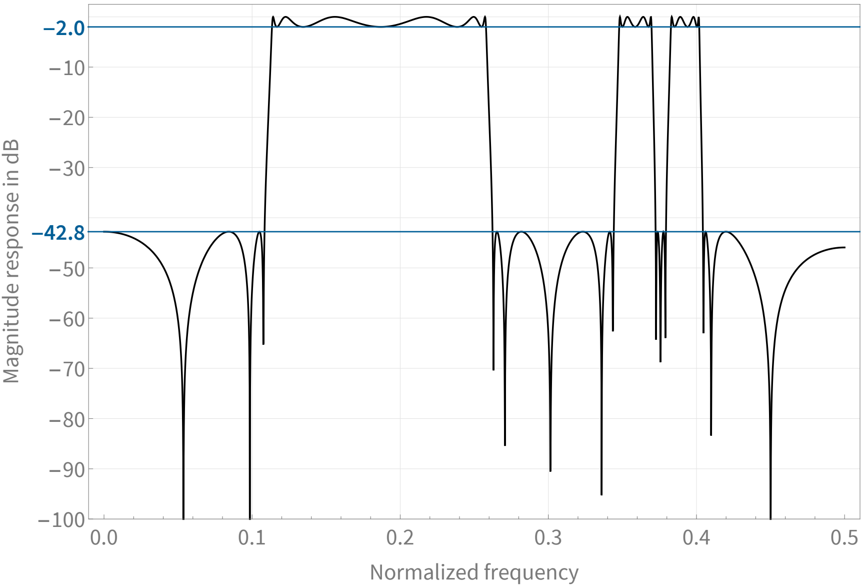
Phase response and group delay



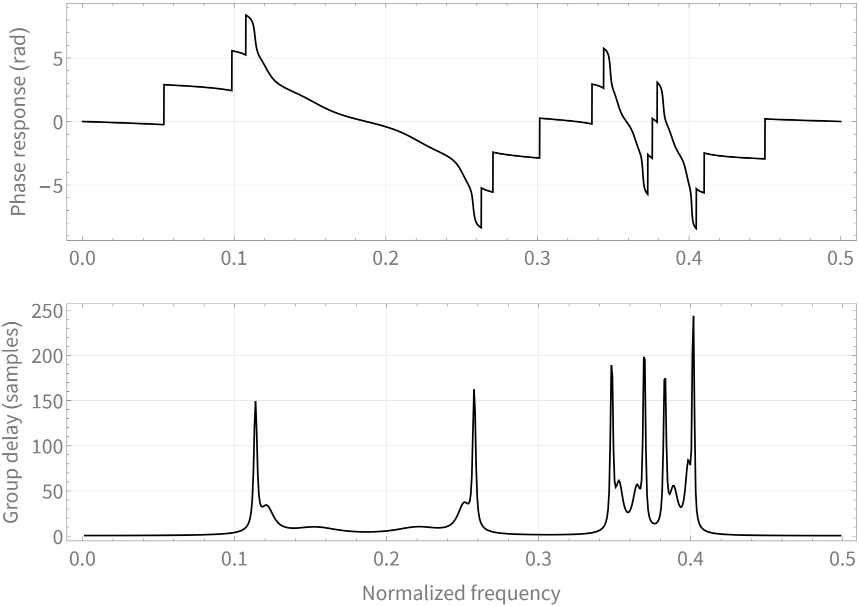
Impulse response (100 and 300 samples)



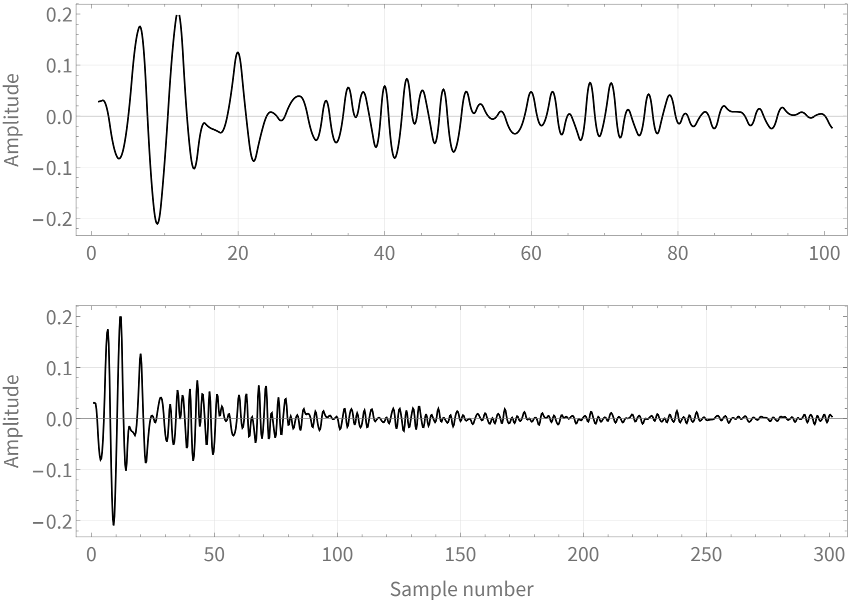
Magnitude response of 28th order optimal filter



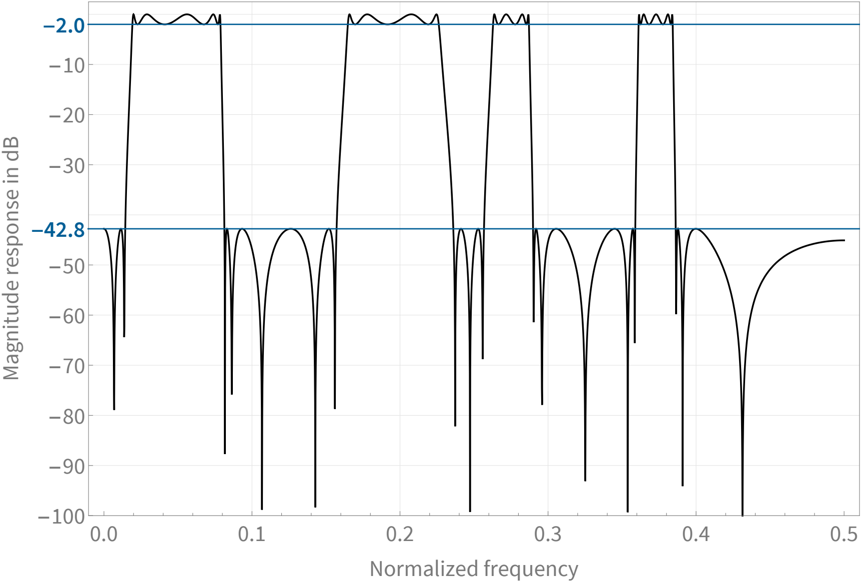
Phase response and group delay



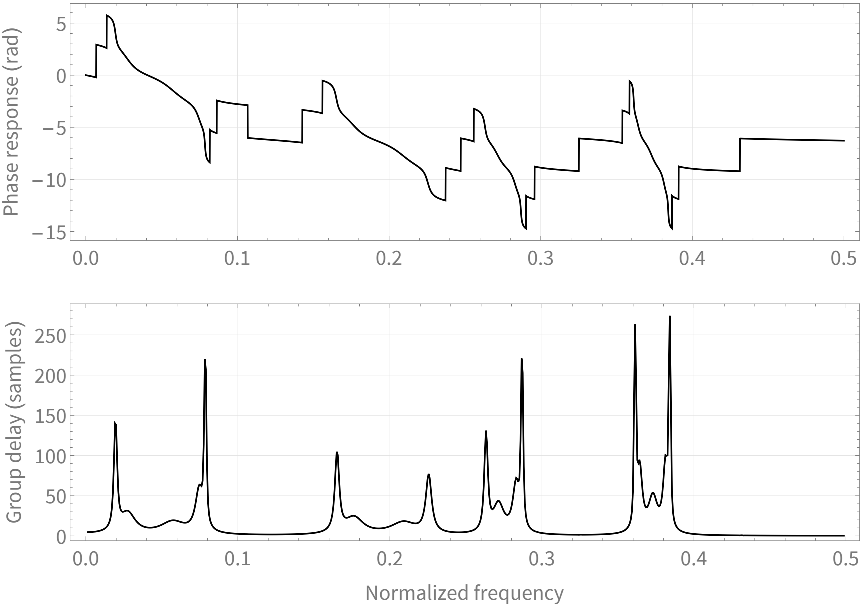
Impulse response (100 and 300 samples)



Magnitude response of 36 th order optimal filter



Phase response and group delay



Impulse response (100 and 300 samples)