Introduction. Noise influence on two-level systems is an important effect in many aspects. It is significant for Josephson-junction qubits. One of the possible ways to solve dissipative problems in such qubits is to consider phases. And geometric phases seemed the most promising. The Aharonov-Anandan phase is a generalization of the Berry phase. Berry showed [1] that phases acquired by nondegenerate eigenstates of a system with slowly varying Hamiltonian can be divided into two parts: dynamic and geometric (Berry phase). The dynamic phase occurs from energy and depends on the time of evolution \( \int_0^T \text{Edt} \), while geometric phase depends only on the geometric properties of the trajectory in the parameter space of the Hamiltonian. However, adiabatic conditions are crucial for this definition, because eigenstates can be determined only in the limit of slowly changing Hamiltonian.

Aharonov and Anandan proposed another definition of the geometric phase [2]. They showed that for arbitrary changes of the Hamiltonian there are states that return to the initial condition but acquire phase, which could be divided into dynamic and geometric (Aharonov-Anandan phase). The dynamic phase is defined as \( \int < \psi|H|\psi > dt \). The geometric phase depends on the trajectory of the state. For a two-level spin-half system in magnetic field it depends only on the solid angle subtended by the spin. There are two states with cyclic evolution for any Hamiltonian change \( H(t) = -\frac{1}{2} B(t) \sigma , g \mu_B = h = 1 \), because evolution operator is unitary, therefore it has two eigenvectors with different eigenvalues. In adiabatic limit we can define eigenstates as spin-vectors along magnetic field \( B \). The difference of their acquired phases is \( \int B dt + \beta (2\text{ from [3]) where } \beta = \int d\phi \cos \theta = 2\pi - W \) is Berry phase (W is a solid angle subtended by the loop \( B(t) \)). Without adiabatic limit there are two opposite spin-vectors with cyclic evolution. The resulting difference of their acquired phases is \( \int (B_n) dt + \beta \) where \( n \) is the unitary vector of the spin direction (spins are collinear but opposite). \( \int (B_n) dt = \int < \psi|H|\psi > dt \) is the dynamic phase and in (2) we show that \( \beta \) has geometric nature. Berry phase is often separated from dynamic in experiments by using spin-echo method, but it cannot be used in general when observed state is not an eigenstate of slowly varying Hamiltonian. For the first time the Aharonov-Anandan phase was measured in the NMR experiment [3].

However, the dynamics of the spin is more complicated because of the coupling with the environment. As interaction leads to the changes of the coherent dynamics of the real system, it is important to estimate the result of this effect in terms of observable parameters. We investigate the influence of the noise on the phases. There was shown [4] that in adiabatic and weak-coupling limit Berry phase has a geometric modification. However, in general phase is more complicated. We calculated it and propose ideas why there is no evident way to divide it into dynamic and geometric parts.

In case of isolated spin we can define difference of phases of cyclic states as an angle acquired by spin orthogonal to \( n \). In the rotating frame where \( z' \)-axis is chosen along \( n \) and \( y' \)-axis orthogonal to original \( z \)-axis (one of the possible ways to define frame which revert to the initial state with \( n \)). Such a frame has the angular velocity \( \omega = \theta t e_y' + \varphi t e_z' \). In this new frame:

\[
\omega = (- \varphi t \sin \theta, \theta t, \varphi t \cos \theta)
\]

and

\[
H_{RF} = U H U^{-1} + i (U_i) U^{-1} = -\frac{1}{2} (\vec{B} + \vec{\omega} ) \hat{\sigma}
\]

We can introduce effective field \( \vec{B}' = \vec{B} + \vec{\omega} = (0, B_z + \omega_z) \) and rewrite the Hamiltonian as \( H_{RF} = -\frac{1}{2} B_z' \sigma_z \) because spin along \( z' \)-axis is static in this frame. Any other spin rotates around \( z' \)-axis with angular velocity \( B_z' \). As a result difference of phases acquired
by two opposite states collinear to \( n \) is
\[
\alpha = \int_0^T B' dt = \int_0^T B_z dt + \int_0^T \omega_z dt. \tag{1}
\]
However, \( \int_0^T B_z dt = \int_0^T < \psi | H | \psi > dt \) and can be called dynamic. Therefore we can define the rest of the
\[
\beta = \int_0^T \omega_z dt = \int d\varphi \cos \theta = 2\pi - W \tag{2}
\]
as geometric (\( W \) is the solid angle subtended by \( z' \).

If we consider coupling with environment as an anisotropic short-correlated (\( \tau_c \ll T \)) noisy field \( \vec{X}(t) = X(t)\hat{e}_z \), the Hamiltonian is
\[
\hat{H} = -\frac{1}{2}B(t)\hat{\sigma}_x + \frac{1}{2}X(t)\sigma_z + H_{env}(X) \tag{3}
\]
In the rotating frame Hamiltonian reads
\[
\hat{H}_{RF} = -\frac{1}{2}B'\sigma_x - \frac{1}{2}X(\cos \theta \sigma_x - \sin \theta \sigma_y) + H_{env} \tag{4}
\]
We can define interaction Hamiltonian as
\[
\hat{H}_{int} = -\frac{1}{2}X(\cos \theta \sigma_x - \sin \theta \sigma_x) \tag{5}
\]
so that \( \hat{H} = \hat{H}_{int} + \hat{H}_0 \).

To observe phases we consider off-diagonal density matrix element, as it contains the phases \( < \sigma_x + i\sigma_y' = \rho_{01} \). Using technique developed by Seoeller and Schön [5,6] we can find kinetic equation for density matrix elements. We obtain equation
\[
\frac{d}{dt}\rho(t) = i [\rho(t), H_0] + \int_0^t dt' \sum S(t - t')\rho_{01}(t') \tag{6}
\]

\( S \) can be defined perturbatively as a sum of irreducible diagrams [5]. As we consider short-correlated noise \( \tau_c \ll T_1, \tau_c \ll T_2 \), where \( T_1 \) is relaxation rate for longitudinal component of spin-vector and \( T_2 \) for transverse (can be considered as energy and phase relaxation rates respectively), we can use Bloch-Redford approximation
\[
\frac{d}{dt}\rho(t) = iB'\rho(t) + \Gamma \rho(t) \tag{6}
\]
where \( \Gamma \) is the Bloch-Redford tensor
\[
\Gamma = \int_0^\infty d\tau \sum \langle \tau | \rho_{01} \rangle | = \int [\rho, H_0] \tag{7}
\]
As we consider weak dissipation, we can use rotating-wave approximation. Therefore for \( \rho_{01} \) we keep contribution only from \( \Gamma_{01-00} \).
\[
\Gamma_{01-00} = -\int dt' S(t - t') \left[ \cos \theta(t) \cos \theta(t') + \frac{1}{2} \sin \theta(t) \sin \theta(t') \exp \left( -i \int t' B(t') dt \right) \right] \tag{8}
\]
where \( S(t - t') = 2 \langle (X(t)X'(t)) + \langle X(t')X(t) \rangle \rangle \).

To the linear order in \( \tau_c \) and \( K\tau_c^2 \)
\[
\Gamma_{01-00} = -\int dt' S(t - t') \left\{ \cos \theta(\cos \theta + \sin \theta \omega_{y'} (t - t')) + \frac{1}{2} \sin \theta (\sin \theta - \cos \theta \omega_{y'} (t - t')) * \exp \left( -i \int (B(t) - K (t - \tau)) dt \right) \right\}
\]
where \( K = \frac{dB'}{dt} \).

For short correlated noise \( S(t) \) is nonzero only on \( \tau < \tau_c \). Therefore we can change integration limits.
\[
\Gamma_{01-00} = -\int dtS(t) \left[ \cos \theta (\cos \theta + \sin \theta \omega_{y'} t) + \frac{1}{2} \sin \theta (\sin \theta - \cos \theta \omega_{y'} t) \exp (-iB't) + \frac{i}{2}\sin^2 \theta K \frac{t^2}{2} \exp (-iB't) \right]
\]
Expressing in terms of Fourier transform of \( S(t) \) we obtain
\[
\Gamma_{01-00} = -i \int \frac{d\Omega}{2\pi} S(\Omega) \left[ \frac{\cos^2 \theta}{(\Omega + i\theta)^2} + \frac{\sin^2 \theta}{2(\Omega - B' + i\theta)^2} \right] + \Omega \omega_{y'} \int \frac{d\Omega}{2\pi} S(\Omega) \left[ \sin \theta \cos \theta (\Omega + i\theta)^2 - \frac{\sin \theta \cos \theta}{2(\Omega - B' + i\theta)^2} \right] - \frac{1}{2}K \sin^2 \theta \int \frac{d\Omega}{2\pi} S(\Omega) \frac{1}{(\Omega - B' + i\theta)^3} \tag{9}
\]
Therefore the phase is
\[
\phi = \int (B' + Im \Gamma_{01-00}) dt
\]
As one can see second and third terms in (9) vanish after being integrated along closed trajectory when \( |B'| \) is
constant (for example, field modulus is constant and it is rotating with constant velocity).

We calculated the total phase change induced by environment influence. However, naïve attempts to single out the dynamic part resulted into nongeometric rest part of the phase. The problem is in nonevident definition of geometric phases in nonadiabatic and noisy case. Common experimental methods such as spin echo or making the system undergo the same evolution within different time intervals (so that dynamic phase varies linearly with time and therefore can be separated from geometric) does not suit for measuring AA phase. Considering real part of Bloch-Redfield tensor one can find dephasing rates [5].

From Bloch equations we get

\[
(r_{01})_t = iB' r_{01} - \frac{1}{T_2} r_{01}
\]

That leads to

\[
\frac{1}{T_2} = -\text{Re} \Gamma_{01\rightarrow01}
\]

And using (9)

\[
\frac{1}{T_2} = \frac{1}{2} S(0) \cos^2 \theta + \frac{1}{4} S(B') \sin^2 \theta - \text{V.p.} \frac{1}{2} \omega \int \frac{d\Omega}{2\pi} S(\Omega) \sin \theta \cos \theta \left( \frac{2}{\Omega^2} - \frac{1}{(\Omega - B')^2} \right) + \text{V.p.} \frac{1}{2} K \sin^2 \theta \int \frac{d\Omega}{2\pi} S(\Omega) \left( \frac{1}{(\Omega - B')^3} \right)
\]

Considering diagonal components of density matrix

\[
(r_{00})_t = -\Gamma_{11\rightarrow00} r_{00} + \Gamma_{00\rightarrow11} r_{11}
\]

\[
(r_{11})_t = \Gamma_{11\rightarrow00} r_{00} - \Gamma_{00\rightarrow11} r_{11}
\]

\[T_1\] is relaxation rate for longitudinal component (energy relaxation rate) \((\langle \sigma_z \rangle = r_{00} - r_{11})\), therefore we obtain

\[
\frac{1}{T_1} = \Gamma_{11\rightarrow00} + \Gamma_{00\rightarrow11} = \frac{\sin^2 \theta}{2} S(B')
\]

We found environment induced correction to the phase of two level system without adiabatic approximation in anisotropic case. That anisotropic case takes place for example in Josephson-junction qubits [6]. And in case of multi-directional coupling we can sum phase modifications calculated for every direction separately.

We calculated dissipation rates , therefore we determined conditions on noise correlation time \(\tau_c\).


